Novel gray coded pattern for unwrapping phase in fringe projection based 3D profiling

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ABSTRACT

A method to reliably extract object profiles even with height discontinuities (that leads to $2n\pi$ phase jumps) is proposed. This method uses Fourier transform profilometry to extract wrapped phase, and an additional image formed by illuminating the object of interest by a novel gray coded pattern for phase unwrapping. Simulation results suggest that the proposed approach not only retains the advantages of the original method, but also contributes significantly in the enhancement of its performance. Fundamental advantage of this method stems from the fact that both extraction of wrapped phase and unwrapping the same were done by gray scale images. Hence, unlike the methods that use colors, proposed method doesn't demand a color CCD camera and is ideal for profiling objects with multiple colors.

Keywords: 3D profile, phase unwrapping, grayscale resolution, Fourier transform profilometry.

1 INTRODUCTION

3D shape measurement finds applications in solid modeling, medical imaging, industry quality control, deformation analysis etc[11]. Fringe projection techniques are preferred because of their non contact nature and the ability to extract complete profile of objects, avoiding problems associated with scanning methods. Extracting 3D profile with captured 2D images is a growing area of interest, one way to achieve the same is through Fourier transform profilometry (FTP) [1,2]. In FTP, a periodic grating is projected on to the surface of a diffusely reflective object. When this grating image is observed under an angle with the projection direction, the grating lines deform due to the object surface shape. Image processing of the captured deformed grating lines reveals the 3D profile of the object of interest. Despite its capability to extract object profiles with single image, FTP fails in reliably extracting 3D profiles of objects with $2n\pi$ (*n* is a nonzero integer) phase jumps. This problem stems from the fact that the grating is periodic, lacking a signature for each period.

Several methods exist to overcome this problem, including temporal phase unwrapping[7], reduced temporal phase unwrapping[8], spatio-temporal phase unwrapping[9] and frequency multiplexing procedures[10]. All these methods require multiple phase maps generated by varying the spatial frequency of projected fringe pattern[3]. A method based on the usage of color bands (one unique color per period) along with the gray bands capable of extracting unwrapped phase with a single phase map exist[3]. However, this method demands a color CCD camera and is not ideal for objects with multiple colors. Current proposal aims at developing a phase unwrapping algorithm that is capable of unwrapping phase of object profiles with height discontinuities with a single gray scale phase map.

2 METHODOLOGY

First step involved in the proposed method is the extraction of wrapped phase using FTP[1,2]. One can use any other methods including (multi channel) phase stepping profilometry[5], spatial phase detection[6] for the extraction of wrapped phase. This article dealt with simulations based on FTP alone.

2.1 Extraction of wrapped phase (FTP)

The intensity profile on the object when illuminated by a sinusoidal grating is given by

$$g(x,y)=a(x,y)+b(x,y)\cos\left(2\pi f_o x+\phi(x,y)\right) \qquad (1)$$

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where $\phi(x, y)$ contains the phase modulated by height of the object and $a(x, y)$ & $b(x, y)$ represents unwanted irradiance variations arising from the non uniform light reflection or transmission by a test object; in most cases, $a(x, y)$, $b(x, y)$ and $\phi(x, y)$ vary slowly compared with the variation introduced by the spatial-carrier frequency f_o.

Eq (1) can be written as

$$g(x, y) = a(x, y) + c(x, y)\mathrm{e}^{2\pi i f_o x} + c^*(x, y)\mathrm{e}^{-2\pi i f_o x} \qquad (2)$$

where

$$c(x, y) = \frac{1}{2} b(x, y) e^{i\phi(x, y)} \qquad (3)$$

Applying Fourier transform to eq (2) with respect to x using Fast Fourier transform (FFT) algorithm results in

$$G(f, y) = A(f, y) + C(f - f_o, y) + C^*(f + f_o, y) \qquad (4)$$

where capital letters denotes the Fourier spectra and f is the spatial frequency in the x direction. Since the spatial variations of $a(x, y)$, $b(x, y)$ and $\phi(x, y)$ are slow compared with the spatial frequency f_o, the Fourier spectra in Eq (4) are separated by the carrier frequency f_o. Either of the two spectra on the carrier, say $C(f - f_o, y)$, can be used by translating the same by f_o on the frequency axis towards the origin to obtain $C(f, y)$. Unwanted background variation $a(x, y)$ has been filtered out in this stage. Using FFT algorithm, inverse Fourier transform of $C(f, y)$ is calculated with respect to f to obtain $c(x, y)$, eq (4). Complex logarithm of the resultant leads to

$$\log\left[c(x, y)\right] = \log\left[\left(\frac{1}{2} b(x, y)\right)\right] + i\phi(x, y) \qquad (5)$$

Imaginary part of eq (5) gives the phase independent of the unwanted amplitude irradiance $b(x, y)$ in the real part. Experimental setup for the same is shown in fig 1. The phase so obtained is indeterminate by a factor of 2π, because of the arctan function used to extract the same. A suitable phase unwrapping algorithm should be used to extract true (unwrapped) phase.

2.2 Phase Unwrapping

Conventional phase unwrapping algorithms works by adding or subtracting 2π relative to the preceding pixel when ever there is a phase jump of 2π between successive pixels[1]. Hence, this unwrapping algorithm fails when the object has height jumps leading to $2n\pi$ phase jumps, as the phase addition is always 2π, not its multiples. True (unwrapped) phase is given by

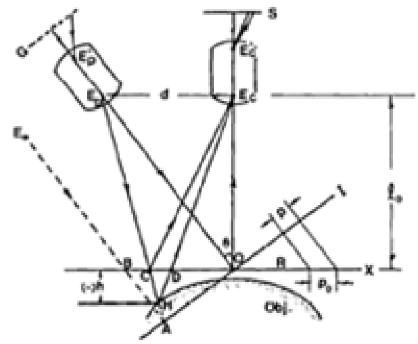
$$\phi_u(x, y) = \phi(x, y) + 2n(x, y)\pi \qquad (6)$$

Hence the challenge of phase unwrapping can be restated as the challenge to find $n(x, y)$.

2.3 Proposed method for phase unwrapping

Along with an image of the deformed grating lines used in FTP, current proposal uses another image, formed by illuminating the object with gray coded pattern. This pattern is an array of uniquely identifiable rectangular bands, arranged in a specific sequence, as shown in Fig 3(a). This pattern is projected on to the reference plane and object of interest. Deviations in the gray value on the object image relative to the reference image at any point on the image correspond to object height deviations.

Width of each band is made exactly equal to the period of the grating used for FTP. Every gray band is made of unique gray level, which acts as a signature to every period. Comparing the gray level of the object and reference gray coded images, one can obtain object phase map reliably even in the presence of surface discontinuities.



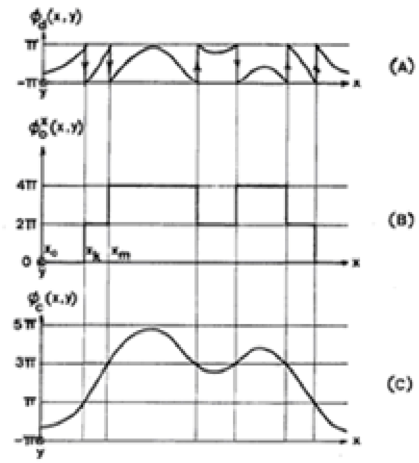


Fig 1: Crossed optical axes geometry. Fig 2: Conventional spatial phase unwrapping

Figure courtesy Takeda et al. [1,2]

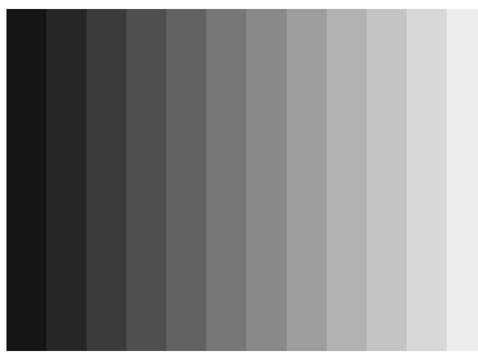


Fig 3(a): Gray coded pattern used for simulations

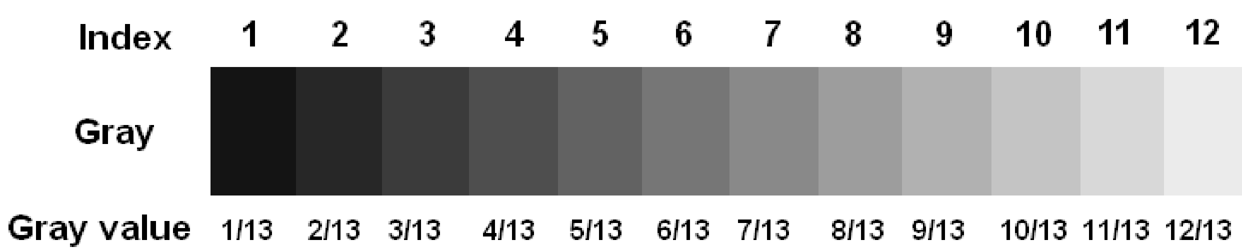


Fig 3(b): Look up table for gray coded pattern

2.4 Limits for the number of gray scales resolvable

As CCD cameras with better than 12-bit tonal range[13] are available commercially, more than 2^{12} gray levels can be discriminated, which facilitates one to use as many as 4096 (grating lines) periods. Scientific cameras with better than 16-bit gray scale resolution are available commercially[12] which leads to 65,536 periods. To overcome the influence of noise, we define B as the threshold to detect noise by requiring that the gray scale difference between successive gray coded bands be larger than B. Stating alternatively; a gray level increment of 1 can not be detected due to noise, whereas an increment of B can be detected. For an n-bit camera, number of recognizable periods H should satisfy

$$H \le 2^n / B \qquad (7)$$

Considering that the noise levels effecting the gray scale resolution are such that an increment of 5 in the gray level can be detected[4], i.e, for B=5 and n=16, $H \leq 13{,}107$, which is sufficient for most of the practical applications. CCD cameras with appropriate gray scale resolution should be used as per requirement.

Table 1: No. of gray levels distinguishable, H, for different n values

n	2^n	*B*	*H*
16	65,536	5	13,107
16	65,536	11	5958
14	16,384	5	3277
14	16,384	11	1490
12	4096	5	819
12	4096	11	372

3 COMPUTER SIMULATIONS

To demonstrate the ability of the proposed method in profiling objects with surface discontinuities, a typical phase object containing three different phase steps of 4π , $\pi/2$ & 2π is considered, Fig 4. Fig 5(a) & 5(b) shows respectively the simulated images of the fringe pattern when the grating shown in Fig 4 is projected on to a reference plane and considered object respectively. Three dimensional surface plot of wrapped phase map extracted using FTP is shown in Fig 6(a). Fig 6(a) is the resultant even after applying spatial phase unwrapping techniques, as the phase jumps are non zero integer multiples of 2π . Since estimating exact number of fringes shifted at each step height is not possible from gray scale fringe pattern (Fig 5(b)) alone, proposed method uses the projection of gray coded pattern too, Fig 7. Fig 7(a) & 7(b) shows the simulated images of the reference plane and object surface respectively when gray coded patterns are projected on them. Ten such bands are present in a window. In order to extract the necessary information from C_o in determining $n(x, y)$, one can apply the algorithm proposed by S S Gorith et al[3]. An alternative procedure is established here, that has the following steps

(1) Extract wrapped phase map using any of the existing methods[1,2,5,6]
(2) Generate Reference gray coded matrix and object gray coded matrix from captured images. A gray coded matrix is nothing but a matrix of gray levels at the corresponding pixel. Image captured when the gray coded pattern is illuminated on the reference/object corresponds to reference/object gray coded matrix.
(3) Prepare Look Up table for indexing bands.
(4) Use the look up table to generate reference index matrix and object index matrix. These matrices are formed by just replacing the gray value in the gray coded matrix with the corresponding index, Fig 3. Also generate wrapped object gray coded matrix, Fig 6(b), by computer simulation using the wrapped phase and look up table.
(5) Generate difference matrix=object index matrix-wrapped object index matrix. Difference matrix directly gives the value of n(x,y), i.e., number of 2π s that should be added to the unwrapped phase at every (pixel) spatial location (x,y).
(6) Extract unwrapped phase by adding the wrapped phase with $2n(x, y)\pi$

Fig 8 shows the unwrapped phase obtained using the proposed algorithm. Fig 9 shows the comparison of wrapped phase and unwrapped phase maps.

Since gray information is used for processing, profiling results can be effected by the gray of the original object. Hence, meaningful results can be obtained only on surfaces with neutral gray contents as is true for any of the previous techniques that use gray[4].

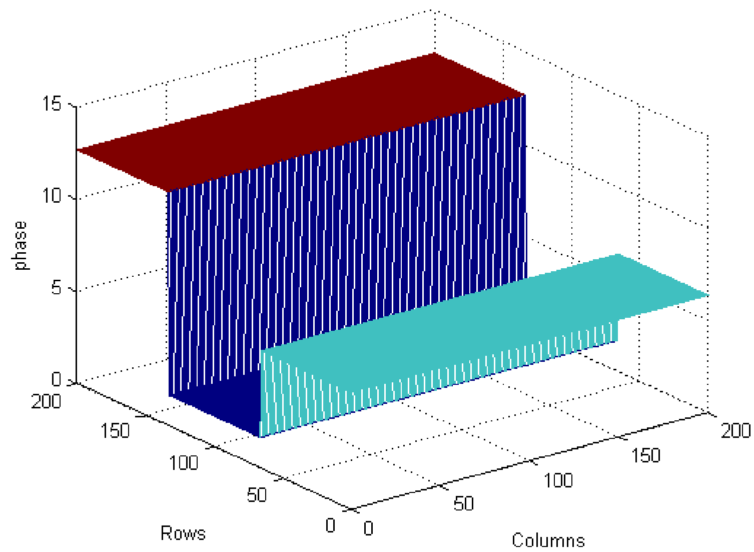


Fig 4: Original phase object considered

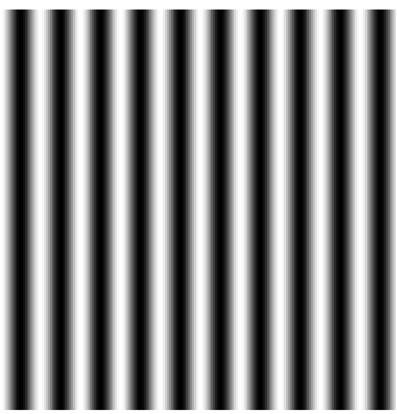


Fig 5(a): Fringe pattern on reference plane

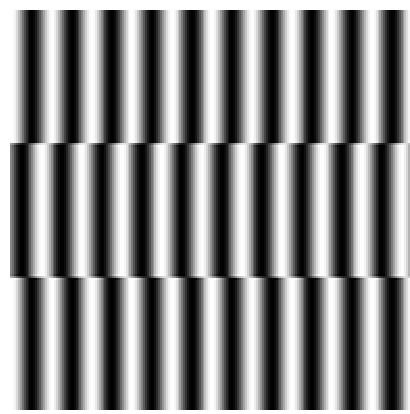


Fig 5(b): Fringe pattern on object considered

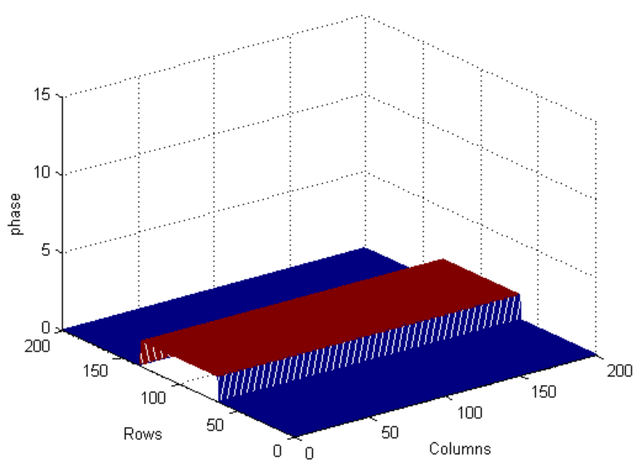


Fig 6(a): Wrapped phase. Same is the result for unwrapping using conventional unwrapping algorithms

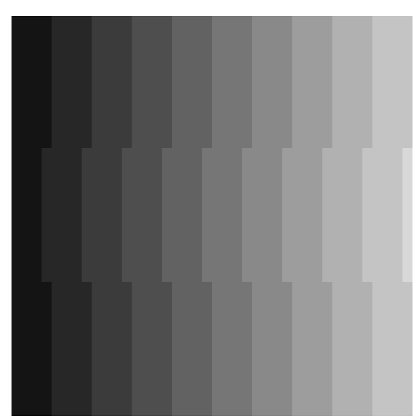


Fig 6(b): Simulated wrapped object gray coded pattern

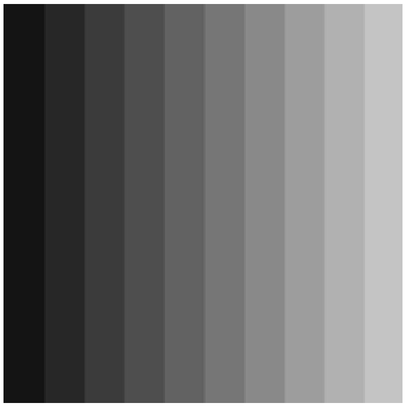


Fig 7(a): Gray coded pattern on reference

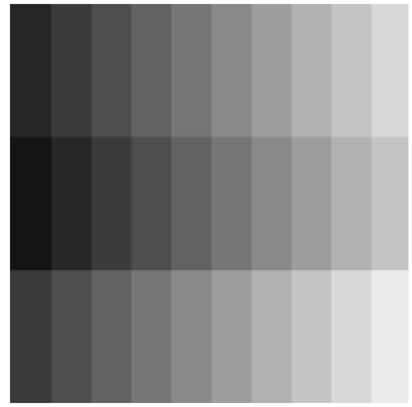


Fig 7(b): Gray coded pattern on object considered

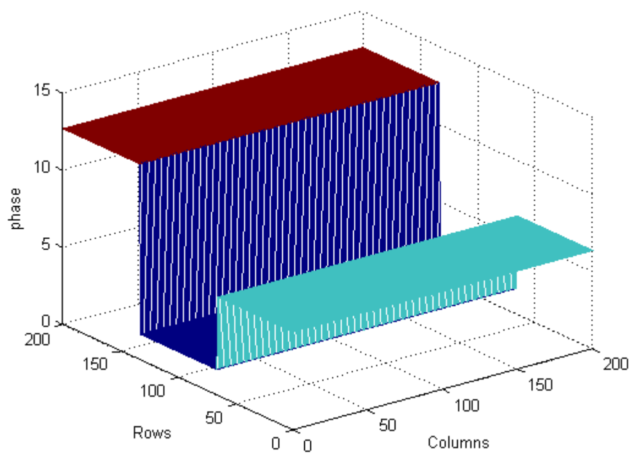


Fig 8: Unwrapping using proposed method

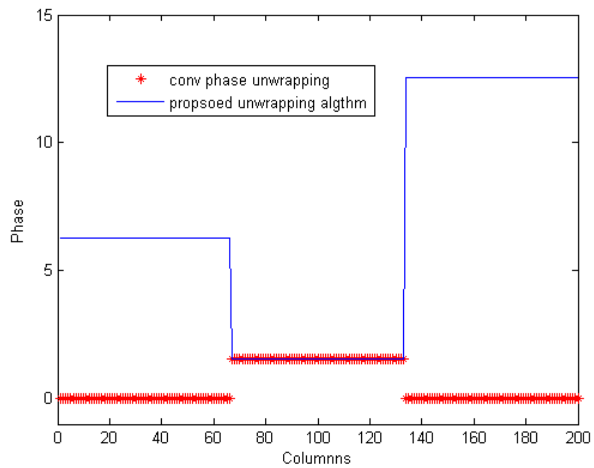


Fig 9: Object profile along 100th column

4 CONCLUSIONS

A novel gray coded pattern that aids in unwrapping phase obtained in fringe projection techniques (FTP, PSP etc) for 3D profiling of objects even with phase jumps is proposed. A procedure for extraction of unwrapped phase is established. Unlike the temporal unwrapping and frequency multiplexing procedures, this new approach requires a single-phase map for surface profile measurement, hence, reducing the number of phase maps required for phase unwrapping by a considerable amount. Hence, proposed method is significantly faster compared to temporal phase unwrapping (which typically demands $\left(\log_2 S+1\right)/2$ images) in terms of the image acquisition and analysis time. Also, unlike the methods based on the usage of colors, this approach doesn't use colors, hence, can be used for objects with multiple colors.

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